Magnetohydrodynamic Flow of a Dusty Rivlin-Ericksen Fluid through Porous Medium Due To an Impulsively Started Plate

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Abstract

In this section we study of a theoretical analysis of two dimensional flow of Rivlin-Ericksen fluid through a porous medium. The unsteady laminar flow of an elastico-viscous fluid suspended with a uniform distribution of non conducting dust particles in presence of uniform transverse magnetic field fixed relative to the plate which is applied perpendicular to the flow of fluid is considered. The velocity field for the conducting dusty fluid, non conducting dust particles and the skin friction at the plate are obtained for large value of time. The effect of magnetic field, porosity and the mass concentration of the dust particle on velocity and skin friction are discussed numerically and graphically.

1. Introduction

Many researchers have worked on flow of a viscous incompressible and electrically conducting fluid over a flat plate in the presence of uniform transverse magnetic fluid which is fixed relative to the plate or to the fluid. The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane, was first studied by Stokes (1851). It is also known as Rayligh's problem in the literature. Following Stokes' analysis, Soundalgekar (1977) first presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. Soundalgekar (1979) has studied mass transfer effects on flow past an impulsively started infinite vertical plate. Rivlin and Erickesn (1955), Happel (1959), Saffman (1962), Baral (1968), Gupta (1960), Choubey (1985), Ram and Mishra (1977), Prakash (1971), Pande (1970), Ahmadi and Manvi (1971), Varshney and Rashmi Kumari (2006) have solved the hydromagnetic flow due to accelerated motion of an infinite flat plate in the presence of magnetic field fixed relative to the plate.

Later a large No. of dusty flow problems have been investigated in the literature and are well documented in a review by Michael and Milless (1966), Michael and Norey (1968) considered the unsteady flow problem of dusty fluid in different ways.

In this section we consider the MHD flow of dusty Rivlin-Ericksen fluid through porous medium due to an impulsively started flat plate.

2. Nomenclature

m	:	Mass of the dust particle
N_0	:	The number density of dust particle
t	:	Time
U	:	Constant velocity of the plate
u, v	:	Velocity component of the conducting dusty elastico viscous liquid and dust particle parallel to the plate respectively
y	:	Rectangular coordinate normal to the plate
υ	:	Kinematic co-efficient of viscosity of the conducting dusty fluid
μ	:	Viscosity of the conducting dusty fluid
σ	:	Electrical conductivity of the conducting dusty fluid
τ	:	The relaxation time of the dust particle
P	:	The density of the conducting dusty fluid
α'	:	Kinematic viscosity
β'	:	The kinematic visco elasticity
l, M	:	Non dimensional magnetic field parameters (Hartmann No.)
K	:	Porosity parameter
K	:	Stokes resistance coefficient
B_0	:	Uniform transverse magnetic field
λ_0	:	The elastic coefficient

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3. Mathematical Analysis

Consider the x-axis along the plate and y-axis normal to it the equations of motion are:-

$$\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+\left(1+\lambda_0\frac{\partial}{\partial t}\right)\frac{kN_0}{\rho}(v-u)$$

$$-\left(1+\lambda_0\frac{\partial}{\partial t}\right)\left(\frac{\sigma B_0^2}{\rho}+\frac{\upsilon}{K}\right)u \qquad ...(3.1)$$

$$and \qquad \tau\frac{\partial v}{\partial t}=(u-v) \qquad .... (3.2)$$

Where $\tau=\dfrac{m}{k}$

Equation (3.1) and (3.2) can be put in non dimensional forms with help of following substitutions

$$t^* = \frac{t}{\tau}$$

$$y^* = \frac{y}{\sqrt{\upsilon\tau}}$$

$$u^* = \frac{u}{U}$$

$$v^* = \frac{v}{U}$$

Then equations (3.1) and (3.2) respectively reduced to

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + l\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u)$$

$$-\left(1+\alpha\frac{\partial}{\partial t}\right)\left(M^2+\frac{1}{K}\right)u \quad ...(3.3)$$

$$and \quad \frac{\partial v}{\partial t} = (u-v) \quad \ldots (3.4)$$

Now when the magnetic field is fixed relative to the plate the eqn. (3.3) will be modified at time $t = 0$, the conducting dusty fluid, the plate and the magnetic field are assumed to be stationary everywhere for all later time the plate and the magnetic field are moving with velocity $u = 1$. Because the magnetic field is moving and the conducting dusty fluid is initially at rest. The relative motion must be accounted for the origin of coordinate system is fixed in space. Hence by transformation of coordinates the eqn. (3.3) becomes

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + l\left(1+\alpha\frac{\partial}{\partial t}\right)(v-u)$$

$$-\left(1+\alpha\frac{\partial}{\partial t}\right)\left(M^2+\frac{1}{K}\right)(u-1) \quad ...(3.5)$$

Eqn. (3.4) and (3.5) are solved under the initial and the boundary conditions are

$$t=0, \quad u=0, \quad v=0 \quad for \quad y>0 \quad \ldots (3.6)$$

$$t>0, \quad u=1 \quad at \quad y=0, \quad u\to finite \quad as \quad y\to\infty \quad \ldots (3.7)$$

Let $\bar{u} = \int_0^\infty u.e^{-st}dt \qquad (s>0)$

$$\bar{v} = \int_0^\infty v.e^{-st}dt \qquad (s>0)$$

be respectively the Laplace transforms of ***u* & *v*** on taking Laplace transforms of (3.5) & (3.4) and using (3.7), we get

$$\frac{\partial^2\bar{u}}{\partial y^2} - p^2\bar{u} = -\frac{1}{s}\left(M^2+\frac{1}{K}\right) \quad \ldots (3.8)$$

and $\bar{v} = \frac{\bar{u}}{(1+s)} \qquad (s<<1) \quad \ldots (3.9)$

Where

$$p^2 = (1+\alpha s)\left[\frac{ls}{(1+s)} + \left(M^2+\frac{1}{K}\right) + s\right] \quad \ldots (3.10)$$

The boundary condition (3.7) transform to

$$\bar{u} = \frac{1}{s} \quad at \quad y=0, \quad \bar{u}\to finite \quad as \quad y\to\infty \quad \ldots (3.11)$$

The solution of (3.8) subject to the condition (3.11) is

$$\bar{u} = \left[\frac{1}{s} - \frac{1}{sp^2}\left(M^2+\frac{1}{K}\right)\right].e^{-py} - \frac{1}{sp^2}\left(M^2+\frac{1}{K}\right) \quad \ldots (3.12)$$

Since the inversion of (3.10) presents some difficulty so we restrict over selves to large value of ***t*.** Now when *t* is large then *s* is very small in this case

$$p = \left[\left\{1+l+\alpha\left(M^2+\frac{1}{K}\right)\right\}s + \left(M^2+\frac{1}{K}\right)\right]^{1/2} \quad \ldots (3.13)$$

Then from (3.12) we get (for large *t*)

$$\bar{u} = \left[\frac{1}{s} - \frac{1}{(s+a)}\right] + \frac{1}{(s+a)}e^{-\left[\left\{1+l+\alpha\left(M^2+\frac{1}{K}\right)\right\}s+\left(M^2+\frac{1}{K}\right)\right]^{1/2}.y} \quad \ldots (3.14)$$

By inversion theorem we get

$$u = \left(1-e^{-at}\right) + e^{-at}.erfc\left[\frac{\sqrt{1+l+\alpha\left(M^2+\frac{1}{K}\right)}.y}{2\sqrt{t}}\right] \quad \ldots (3.15)$$

$$where \quad a = \frac{\left(M^2+\frac{1}{K}\right)}{\left\{1+l+\alpha\left(M^2+\frac{1}{K}\right)\right\}}$$

Similarly taking inverse transform for large value of *v* we get from (3.9)

$$v = 1-\left(\frac{a}{a-1}\right).e^{-t} + \frac{e^{-t}}{(a-1)}.erfc\left\{\frac{\sqrt{1+l+\alpha\left(M^2+\frac{1}{K}\right)}.y}{2\sqrt{t}}\right\}$$

$$+\frac{e^{-at}}{(a-1)}\left[1-erfc\left\{\frac{\sqrt{1+l+\alpha\left(M^2+\frac{1}{K}\right)}.y}{2\sqrt{t}}\right\}\right] \quad ...(3.16)$$

Equations (3.15) and (3.16) represent the velocities of the conducting dusty elastico-viscous liquid and dust particles for large *t* in the case when the magnetic field is fixed relative to the plate.

If we put $\lambda_0 = 0$ the all the results agree with the results obtained by Mitra (1985).

4. Calculation of Skin Friction

Shearing stresses at the plate where t is large is given by

$$\left(-\frac{\partial u}{\partial y}\right)_{y=0} = \sqrt{\frac{1+l+\alpha\left(M^2+\frac{1}{K}\right)}{\pi t}}.e^{-at} \quad \ldots (4.1)$$

5. Results and Discussion

In order to get a physical understanding of the problem which are generally valid for large value of t numerical calculations are carried out for the velocities of the conducting dusty liquid non conducting dust particles and the friction at the plate from eqns. (3.15), (3.16) and (4.1) respectively.

The velocity distribution of the conducting dusty fluid and non conducting dust particles are plotted in figures – (7.1) to (7.2) respectively for different value of M, K and l at different points of the flow field. It is noted that for fixed $t = 2$, $\alpha = 2.0$ the velocity of the conducting dusty fluid and non conducting dust particles gradually decreases with the increasing values of M and l, but the velocities increases with the increasing value of K.

The skin friction at the plate is plotted in figure - (7.3) for different value of M, l, t and K. It is noted that for fixed $\alpha = 2.0$, the skin friction decreases continuously with increasing in time t. It is observed that the skin friction decreases with the increasing value of K, but the skin friction increases with the increasing values of M and l.

6. Conclusions

The theoretical analysis for two dimensional flow of Rivlin-Ericksen fluid through a porous medium. The unsteady laminar flow of an elastico-viscous fluid suspended with a uniform distribution of non conducting dust particles in presence of uniform transverse magnetic field fixed relative to the plate which is applied perpendicular to the flow of fluid is considered. The solutions are in terms of exponential functions. The study concludes the following results;

1 The velocity of the conducting dusty fluid and non conducting dust particles gradually decreases with the increasing values of M and l, but the velocities increases with the increasing value of K.

2 The skin friction decreases with the increasing value of K, but the skin friction increases with the increasing values of M & l.

7. Graphs

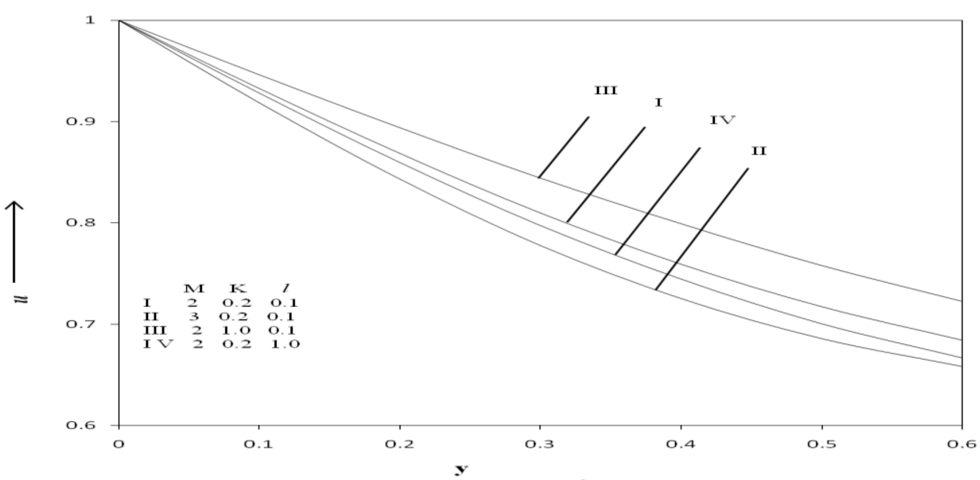

Fig.- (7.1) The velocity profile of fluid for different value of M, K and l.

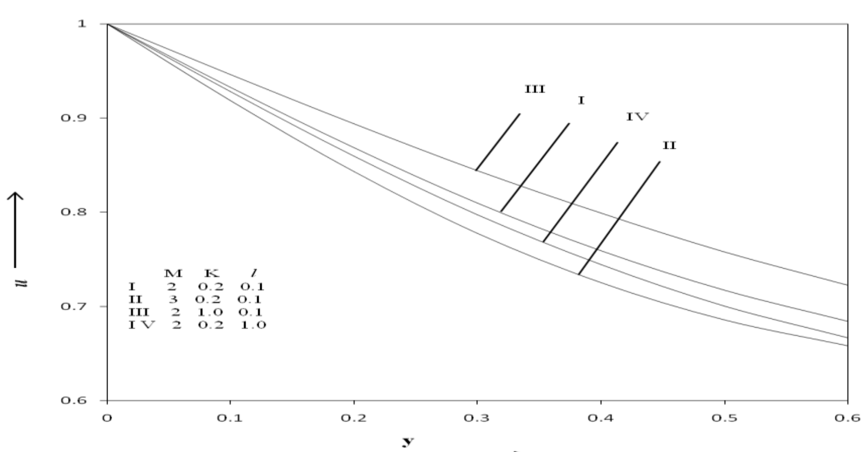

Fig.- (7.1) The velocity profile of fluid for different value of M, K and l.

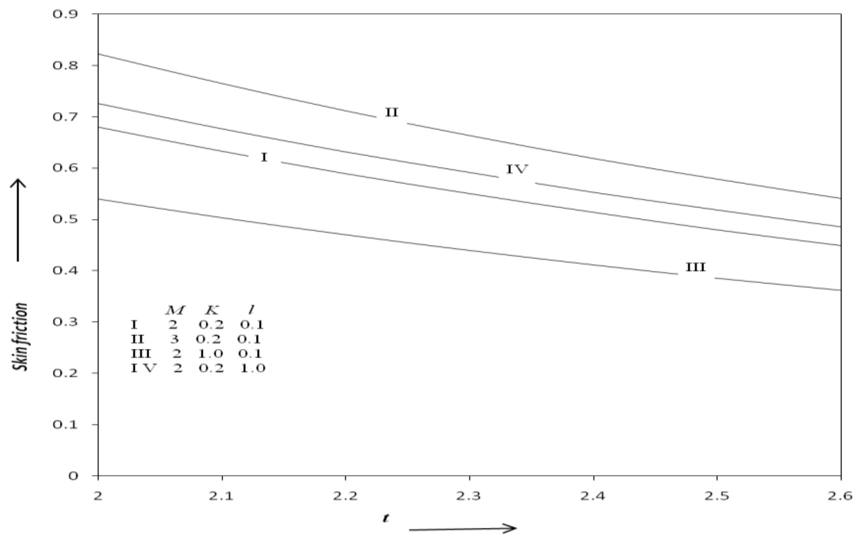

Fig.-(7.3). The skin friction at the plate for different value of M, K and l.

References

[1] G. Ahmadi, R. Manvi, Indian J. Tech., 9, 441, 1971
[2] M. C. Baral, J. phys. Japan, 25, 1701, 1977
[3] K. R. Choubey, Indian Jour. Pure and Appl. Math, 19(8), 931, 1985
[4] A. S. Gupta, *J. Phys. Soc. Japan*, 15, 1894, 1960
[5] J. Happel, J. Am. Inst. Eng., 5, 174, 1959
[6] G. C. Pande,: Ref. Sci., J., 20, 135, 1970
[7] S. Prakash, Indian Journals of Pure and Appl. Math., 2, 108, 1972
[8] R. S. Rivlin, J. L. Erickesn, Stress deformation relation for isotropic material. J. Rat. Mech. Anul., 4, 323-425, 1955
[9] G. Ram, K. S. Mishra, Indian Journals of Pure and Appl. Math., 8, 637, 1977
[10] P. G. Saffman, J. of Fluid Mech., 13, 120, 1962
[11] V. M. Soundalgekar, *ASME. J. heat transfer*, 99, 499-501, 1977
[12] V. M. Soundalgekar, ASME. J. Appl. Mech. 46, 757-760, 1979
[13] G. G. Stokes, Camber. Phil. IX, 8-106, 1851
[14] D. H. Michall, D. A. Milles, Mathmatica, 13, 197, 1966
[15] D. H. Michael, P. W. Norey, Quart Jour. Mech. Appl. Math., 21, 375, 1968
[16] N. K. Varshney, Rashmi Kumari, Acta Ciencia Indica, 32(1), 301, 2006